

# On Minimal Subspaces in Tensor Representations

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**Abstract** In this paper we introduce and develop the notion of minimal subspaces in the framework of algebraic and topological tensor product spaces. This mathematical structure arises in a natural way in the study of tensor representations. We use minimal subspaces to prove the existence of a best approximation, for any element in a Banach tensor space, by means of a tensor given in a typical representation format (Tucker, hierarchical, or tensor train). We show that this result holds in a tensor Banach space with a norm stronger than the injective norm and in an intersection of finitely many Banach tensor spaces satisfying some additional conditions. Examples using topological tensor products of standard Sobolev spaces are given.

**Keywords** Numerical tensor calculus · Tensor product · Tensor Banach space · Minimal subspace · Weak closedness

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