INFINITY MANIFOLDS OF CUBIC POLYNOMIAL HAMILTONIAN VECTOR FIELDS WITH 2 DEGREES OF FREEDOM

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Let X be the Hamiltonian vector field with two degrees of freedom associated to the cubic polynomial Hamiltonian $H(x,y,z,w)$. Using the Poincaré compactification we show that all the energy levels of X in R^4 reach the infinity in a surface topologically equivalent to the intersection of the 3-dimensional sphere $S^3 = \{(x,y,z,w) \in \mathrm{R}^4 : x^2+y^2+z^2+w^2=1\}$ with $\{(x,y,z,w) \in \mathrm{R}^4 : H_3(x,y,z,w)=0\}$, where H_3 denotes the homogeneous part of degree 3 of H. Such a surface is called the *Infinity Manifold* associated to H. In this paper we describe all possible infinity manifolds of cubic polynomial Hamiltonian vector fields with 2 degrees of freedom. Our method is general, but since actual computations can become very cumbersome, we work out in detail only three out of ten possible cases.

1 Introduction

We consider the Hamiltonian vector field X with 2 degrees of freedom generated by a cubic polynomial in four variables $H(x,y,z,w)$. This vector field can be extended to infinity in the following way. Poincaré compactification allows to project the vector field onto the north and the south hemispheres of the sphere $S^4 \subset \mathbb{R}^5$. The extended vector field $\widetilde{X}$ on the equator $S^3 \subset S^4$ represents the asymptotic behavior at infinity of our original Hamiltonian vector field. The vector field in the open hemispheres is not Hamiltonian any more but it keeps having nevertheless a first integral. All the corresponding energy levels intersect the equator S^3 in the same (perhaps singular) 2 dimensional infinity surface W, invariant by $\widetilde{X}$. This surface turns out to be exactly the intersection with the sphere S^3 of the set of zeroes of the cubic homogeneous part $H_3(x,y,z,w)$ of $H(x,y,z,w)$. We see that W is compact