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ON THE PARAMETER DEPENDENCE OF THE REAL CUBIC SURFACES IN ARNOLD'S FORM

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We consider the topological classification of cubic surfaces which are obtained as intersection of the sphere $\mathbb{S}^3$ with the algebraic variety defined by the zeroes of a homogeneous cubic polynomial in Arnold's normal form. This classification is based on the parameters appearing in this normal form, obtaining a correspondence between the parameters of the surface and its topological type. General classifications of cubic surfaces are made in the projective space $\mathbb{P}^3(\mathbb{R})$, but our method, based on a very simple combinatorial procedure is easier to implement in $\mathbb{S}^3$. We split the cubic surfaces parameter space into ten equivalence classes.

1. Introduction.

This work is concerned with the relation between coefficients of the cubic surface $W = W(a, b, c, d) = \{x^3 + y^3 + z^3 + w^3 + (ax + by + cz + dw)^3 = 0\} \bigcap \mathbb{S}^3$ and its topological type. The first studies of the cubic surfaces in $\mathbb{P}^3(\mathbb{C})$ date from 1849, when Cayley and Salmon proved their result that each cubic surface contains at most 27 straight lines. The classification problem of cubic surfaces in $\mathbb{P}^3(\mathbb{C})$ was considered by Schläfli [11] in 1864 obtaining five types. Schläfli [11] , Klein [7] and Rodenberg [9] classified in the XIX century the cubic surfaces in $\mathbb{P}^3(\mathbb{R})$ with some minor gaps. Segre [12] in 1941 wrote the most complete work on the cubic surfaces in $\mathbb{P}^3(\mathbb{C})$ and $\mathbb{P}^3(\mathbb{R})$. He summarized all the known results until that moment. A modern approach was considered by Silhol [13]. The singular cubic surfaces have been studied again in 1978

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