n-1 Independent First Integrals for Linear Differential Systems in \mathbf{R}^n and \mathbf{C}^n

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We prove that every linear system with constant coefficients on \mathbb{R}^n or \mathbb{C}^n is Darboux integrable by providing a complete explicit list of n-1 independent Darboux first integrals.

Key Words: First integral, Darboux integrability, invariant surface

1. INTRODUCTION

Integrability is a topic of great interest in the study of differential equations. We take this notion to mean that a system of differential equations has a sufficient number of independent first integrals belonging to some specified class. But, we note that the term "integrable" is used in several other ways (see [6] and [14]). The purpose of this paper is to show that every *n*-dimensional linear system with constant coefficients is Darboux integrable in the sense that n - 1 independent explicit Darboux integrals can be constructed.

Darboux integrals have been studied extensively for polynomial differential systems. Darboux constructed first integrals for planar polynomial

233