ZERO–HOPF BIFURCATION IN 2–DIMENSIONAL PREDATOR–PREY MODELS

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Abstract. We study the competition between two species according the following modification of the Holling–Tanner II model

$$x' = x\left[r\left(1 - \frac{x}{K}\right) - \frac{qy}{x^2 + a}\right],$$
$$y' = sy\left(1 - \frac{y}{nx + c}\right).$$

Of course, $x \geq 0$, $y \geq 0$ and the parameters a, c, K, n, q, r and s are positive. We prove that its unique positive equilibrium point never exhibits a classical Hopf bifurcation, but for convenient values of the parameters from this equilibrium point bifurcates a periodic orbit, and during this local bifurcation the eigenvalues of such equilibrium remain purely imaginary.

1. Introduction and statement of the main results

The differential system

$$\begin{aligned} x' &= x\left[r\left(1 - \frac{x}{K}\right) - \frac{qy}{x + a}\right], \\ y' &= sy\left(1 - \frac{y}{nx}\right), \end{aligned} \tag{1}$$

is a modification of the classical model of May (see [1, 3, 7, 11]) also known as the Holling–Tanner model (see [2, 5, 6]). Here the variables x, y and the parameters a, K, n, q, r and s are positive. As usual the prime denotes derivative with respect to the time t.

In the differential system (1) we have that $x(t)$ and $y(t)$ denote the prey and predator densities, respectively, as functions of the time t. Moreover, the parameters have the following meanings:

(i) q is the maximal predator per capita consumption rate, in other words the maximum number of prey that can be eaten by a predator in each unit of time.

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