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On the calibration of a Gaussian Heath–Jarrow–Morton model using consistent forward rate curves

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In this paper we propose a calibration algorithm, using a consistent family of yield curves, that fits a Gaussian Heath–Jarrow–Morton model jointly to the implied volatilities of caps and zero-coupon bond prices. The calibration approach is evaluated in terms of in-sample data fitting as well as stability of parameter estimates. Furthermore, the efficiency is tested against a non-consistent traditional method using simulated and market data. Also, we discuss the convergence of the algorithm by means of Monte Carlo simulations.

Keywords: Calibration of deterministic volatility; Control of stochastic systems; Derivatives pricing; Fixed income derivatives

1. Introduction

Any acceptable model that prices interest rate derivatives must fit the observed term structure. This idea, pioneered by Ho and Lee (1986), has been explored in the past by many other researchers, including Black and Karasinski (1991) and Hull and White (1990).

Contemporary models are more complex because they consider the evolution of the whole forward curve as an infinite system of stochastic differential equations (Heath *et al.* 1992) (HJM). In particular, they use a continuous forward rate curve as initial input. In reality, one only observes a discrete set composed either of bond prices or swap rates. Therefore, in practice, the usual approach is to interpolate the forward curve using splines or other parametrized families of functions.

A very plausible question arises at this point: Choose a specific parametric family, $\mathcal{G}$, of functions that represent the forward curve, and also an arbitrage-free interest rate model $\mathcal{M}$. Assume that we use an initial curve that lies within the input for model $\mathcal{M}$. Will this interest rate model evolve through forward curves that lie within the family? Motivated by this question, Björk and Christensen (1999) define the so-called consistent pairs $(\mathcal{M}, \mathcal{G})$ as those whose answer to the above question is positive. In particular, they studied the problem of consistency between the family of curves proposed by Nelson and Siegel (1987) and any HJM interest rate model with deterministic volatility, concluding that there is no such interest model consistent with it.

We remark that the Nelson and Siegel interpolating scheme is an important example of a parametric family of forward curves, because it is widely adopted by central banks (see, for instance, BIS 2005). Its forward curve shape, $G_{\mathrm{NS}}(z, \cdot)$, is given by the expression

$$G_{\mathrm{NS}}(z, x) = z_1 + z_2 \, \mathrm{e}^{-z_4 x} + z_3 x \, \mathrm{e}^{-z_4 x},$$

where x denotes time to maturity and z the parameter vector

$$z = (z_1, z_2, \ldots).$$

Despite all the positive empirical features and general acceptance by the financial community, Filipovic (1999) has shown that there is no Itô process that is consistent with the Nelson–Siegel family. In a recent study, De Rossi (2004) applies consistency results to propose a consistent

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