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A Proper Generalized Decomposition for the solution of elliptic problems in abstract form by using a functional Eckart–Young approach

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ABSTRACT

The Proper Generalized Decomposition (PGD) is a methodology initially proposed for the solution of partial differential equations (PDE) defined in tensor product spaces. It consists in constructing a separated representation of the solution of a given PDE. In this paper we consider the mathematical analysis of this framework for a larger class of problems in an abstract setting. In particular, we introduce a generalization of Eckart and Young theorem which allows to prove the convergence of the so-called progressive PGD for a large class of linear problems defined in tensor product Hilbert spaces.

1. Introduction

The Proper Generalized Decomposition (PGD) method has been recently proposed [1,15,19] for the *a priori* construction of separated representations of an element u in a tensor product space $V = V_1 \otimes \cdots \otimes V_d$, which is the solution of a problem

$$A(u) = l. \tag{1}$$

A rank-n approximated separated representation u_n of u is defined by

$$u_n = \sum_{i=1}^{n} v_i^1 \otimes \cdots \otimes v_i^d, \tag{2}$$

with $v_i^k \in V_k$ for $1 \leqslant i \leqslant n$ and $1 \leqslant k \leqslant d$. The *a posteriori* construction of such tensor decompositions, when the function u is known, have been extensively studied over the past years in multilinear algebra community [6,7,13,14,4,8] (essentially for finite-dimensional vector spaces V_i). The question of finding an optimal decomposition of a given rank r is not trivial and has led to various definitions and associated algorithms for the separated representations.

In the context of problems of type (1), the solution is not known *a priori*, nor an approximation of it. An approximate solution is even unreachable with traditional numerical techniques when dealing with high dimensions d. It is the so-called curse of dimensionality associated with the dramatic increase of the dimension of approximation spaces when increasing d. The PGD method aims at constructing a decomposition of type (2) without knowing *a priori* the solution u. The aim of the PGD is to construct a sequence u_n based on the knowledge of operator A and right-hand side l. This can be achieved

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