NUMERICAL STUDY OF THE GEOMETRY OF THE PHASE SPACE OF THE AUGMENTED HILL THREE-BODY PROBLEM

Ariadna Farrés*, Àngel Jorba†, and Josep-Maria Mondelo‡

February 23, 2017

Abstract

The Augmented Hill Three-Body problem is an extension of the classical Hill problem that, among other applications, has been used to model the motion of a solar sail around an asteroid. This model is a 3 degrees of freedom (3DoF) Hamiltonian system that depends on four parameters. This paper describes the bounded motions (periodic orbits and invariant tori) in an extended neighbourhood of some of the equilibrium points of the model. An interesting feature is the existence of equilibrium points with a 1:1 resonance, whose neighbourhood we also describe. The main tools used are the computation of periodic orbits (including their stability and bifurcations), the reduction of the Hamiltonian to centre manifolds at equilibria, and the numerical approximation of invariant tori. It is remarkable how the combination of these techniques allows the description of the dynamics of a 3DoF Hamiltonian system.

1 Introduction

In the recent years, a lot of attention has been devoted to the study of asteroids and comets. On one side, they carry information about the past of the Solar System. On the other hand, we are starting to look at them as potential resources of raw minerals and volatiles, to be used in-situ for space exploration, or to be sent back to Earth. Moreover, the so-called Near-Earth Objects (NEOs) are also of high interest due to the risk of collisions with the Earth.

Solar sailing is a novel way of navigating an unmanned spacecraft in the Solar System, and it could allow multi-rendezvous missions to visit several asteroids [DBB+14]. It is remarkable that, as the gravitational field of an asteroid is small, solar radiation pressure becomes relevant when moving around them [VRF+12]. For this reason, the use of the solar radiation pressure to navigate around asteroids can give rise to new mission concepts.

The three dominant effects for a small solar sail moving near an asteroid are the gravitational attraction of the asteroid, the gravitational field from the Sun and the solar radiation pressure (SRP). The first effect also includes the shape and rotation of the asteroid, which are relevant within a few radii from its surface (this is known as the gravity regime). If the distance to the asteroid is larger, it is enough to consider it as a point mass. This is the case in which the solar gravitation and SRP are relevant and must be considered. Moreover, as the distance to the Sun is much larger than the distance to the asteroid, Sun's gravity and SRP can be taken as uniform forces. For this reason, we use as a model for the dynamics the Augmented Hill 3-body problem, AH3BP, [MSL01, GF12, FJMV14], which is the classical Hill problem [Hil78, Sze67] with an additional term that accounts for the effect of the solar sail. This term depends on the sail's efficiency,

*Postdoc Researcher, Institut de Matemàtiques de la Universitat de Barcelona, Universitat de Barcelona, Gran Via de les Corts Catalanes 585, 08007 Barcelona, Spain

†Professor, Departament de Matemàtiques i Informàtica, Universitat de Barcelona, Gran Via de les Corts Catalanes 585, 08007 Barcelona, Spain

‡Associate Professor, IEEC & Departament de Matemàtiques, Universitat Autónoma de Barcelona, Bellaterra 08193 (Barcelona), Spain.