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## Periodic solutions for a class of Duffing differential equations

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ABSTRACT. We provide sufficient conditions for the existence of periodic solutions for the class of Duffing differential equations

$$x'' + c(t)x' + a(t)x + b(t)x^{3} = h(t, x),$$

where the functions a(t), b(t), c(t) and h(t, x) are  $C^2$  and *T*-periodic in the variable *t*. **2010 Mathematics Subject Classification**.34C15, 34C25, 37G15, 37C80, 37C30. **Key words and phrases.** Periodic solution; averaging method; Duffing differential equation.

## 1. Introduction and statement of the main results

Several classes of Duffing differential equations have been investigated by many authors. They are mainly interested in the existence of periodic solutions, in their multiplicity, stability, bifurcations, ... see the survey of Mawhin [12], and also the articles [4]–[8], [13, 14, 17, 19] without trying to be exhaustive in the references.

In this work we study the periodic solutions for the class of Duffing differential equations

$$x'' + c(t)x' + a(t)x + b(t)x^{3} = h(t, x),$$
(1.1)

where the functions a(t), b(t), c(t) and h(t, x) are  $C^2$  and T-periodic with respect to the variable t. As usual here the prime denotes derivative with respect to the time t.

In [18] the authors used the Leray–Schauder method to prove the existence of periodic solutions of the Duffing equation

$$x'' + cx' + g(t, x) = h(t),$$
(1.2)

where c > 0 is fixed, *h* and *g* are *T*-periodic functions in *t*.

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