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A class of polynomial planar vector fields with polynomial first integral ☆

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ABSTRACT

We give an algorithm for deciding whether a planar polynomial differential system has a first integral which factorizes as a product of defining polynomials of curves with only one place at infinity. In the affirmative case, our algorithm computes a minimal first integral. In addition, we solve the Poincaré problem for the class of systems which admit a polynomial first integral as above in the sense that the degree of the minimal first integral can be computed from the reduction of singularities of the corresponding vector field.

1. Introduction

In this paper we are concerned with planar polynomial differential systems. One of the main open problems in their qualitative theory is to characterize the integrable ones. The importance of the first integral is in its level sets: such a function H whereas it is defined determines the phase portrait of the system, because the level sets $H = h$ give the expression of the solution curves laying on the domain of definition of H. Notice that when a differential equation admits a first integral, its study can be reduced in one dimension. In addition, Prelle and Singer [46], using methods of differential algebra, showed that if a polynomial vector field has an elementary first integral, then it can be computed using Darboux theory of integrability [24], and Singer [49] proved that if it has a Liouvillian first integral, then it has integrating factors given by Darbouxian functions [20]. Consequently, given a planar differential system, it is important

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