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# On the computation of Darboux first integrals of a class of planar polynomial vector fields ☆

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A B S T R A C T

We study the class of planar polynomial vector fields admitting Darboux first integrals of the type $\prod_{i=1}^{r} f_i^{\alpha_i}$, where the $\alpha_i$'s are positive real numbers and the $f_i$'s are polynomials defining curves with only one place at infinity. We show that these vector fields have an extended reduction procedure and give an algorithm which, from a part of the extended reduction of the vector field, computes a Darboux first integral for generic exponents.



## 1. Introduction

Complex planar polynomial differential systems are being studied since the 19th century when Darboux [18], Poincaré [41,42], Painlevé [39] and Autonne [5] significantly contributed to this topic. Surprisingly, nowadays, the problem of characterizing integrable differential systems as above remains open. To compute a first integral is a very interesting issue because this function provides the solution curves of the system within their domain of definition, determining the phase portrait of the system.

Darboux functions are a remarkable family of multi-valued functions. They have the following shape:

$$H := \prod_{i=1}^{p} f_i^{\lambda_i} \prod_{j=1}^{q} \exp\left(\frac{h_j}{g_j}\right)^{\mu_j}, \tag{1}$$

where $f_i$, and $g_j$ and $h_j$ are bivariate complex polynomials and $\lambda_i$ and $\mu_j$ complex numbers.

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