# NEW CENTRAL CONFIGURATIONS OF THE $(n+1)$–BODY PROBLEM

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Abstract. In this article we study central configurations of the $(n+1)$–body problem. For the planar $(n+1)$–body problem we study central configurations performed by $n \geq 2$ bodies with equal masses at the vertices of a regular $n$–gon and one body with null mass. We also study spatial central configurations considering $n$ bodies with equal masses at the vertices of a regular polyhedron and one body with null mass.

## 1. Introduction and statement of the main results

The $N$–*body problem* consists in study the motion of $N$ pointlike masses in $\mathbb{R}^2$ or $\mathbb{R}^3$, interacting among themselves through no other forces than their mutual gravitational attraction according to Newton's gravitational law.

The *equations of motion* of the $N$–body problem are

$$m_i \ddot{r}_i = -\sum_{\substack{j=1 \\ j \neq i}}^{N} \frac{m_j m_i}{r_{ij}^3}(r_i - r_j),$$

for $i = 1, \ldots, N$. Here we have chosen the units of length in order that the gravitational constant be equal to one, $r_k \in \mathbb{R}^2$ or $\mathbb{R}^3$ is the position vector of the punctual mass $m_k$ in an inertial system, and $r_{jk} = |r_j - r_k|$ is the Euclidean distance between $m_j$ and $m_k$.

Since the general solution of the $N$–body problem cannot be given, great importance has been attached from the very beginning to the search for particular solutions where the $N$ mass points fulfilled certain initial conditions. Thus a *homographic solution* of the $N$–body problem is a solution such that the configuration formed by the $N$–bodies at the instant $t$ remains similar to itself as $t$ varies.

Two configurations are *similar* if we can pass from one to the other doing a dilation and/or a rotation.

The first three homographic solutions where found in 1767 by Euler [8] in the 3–body problem. For these three solutions the configuration of the 3 bodies is collinear.

In 1772 Lagrange [14] found two additional homographic solutions in the 3–body problem. Now the configuration formed by the 3 bodies is an equilateral triangle.

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