Acta Appl Math (2015) 139:167–186
DOI 10.1007/s10440-014-9974-0

Seeking Darboux Polynomials

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Received: 3 July 2014 / Accepted: 22 September 2014 / Published online: 4 October 2014

Abstract We introduce several techniques which allow to simplify the expression of the cofactor of Darboux polynomials of polynomial differential systems in $\mathbb{R}^n$. We apply these techniques to some well-known systems when $n = 2, 3, 4$. We also propose a general method for computing Darboux polynomials in the plane. As an application we prove that a family of potential systems, that includes the van der Pol one, has no Darboux polynomials, giving in particular a new simple proof that the van der Pol limit cycle is not algebraic.

Keywords Planar polynomial differential system · Darboux polynomial · Cofactor · Birational map · Non-algebraic limit cycle

Mathematics Subject Classification (2010) 34C05 · 34A34 · 34C14

1 Introduction and Statement of the Main Results

Consider the polynomial autonomous differential system of $\mathbb{R}^n$ of degree $d \in \mathbb{N}$,

$$\dot{\mathbf{x}} = \frac{d\mathbf{x}}{dt} = P(\mathbf{x}) = \big(P_1(\mathbf{x}), \ldots, P_n(\mathbf{x})\big), \tag{1.1}$$

where $\mathbf{x} = (x_1, \ldots, x_n)$, $P_1, \ldots, P_n \in \mathbb{R}[\mathbf{x}]$ are coprime polynomials of degree at most $d = \max\{\deg P_1, \ldots, \deg P_n\}$ and $t \in \mathbb{R}$ is the independent variable. We denote by $X(\mathbf{x}) = \sum_{i=1}^{n} P_i(\mathbf{x}) \frac{\partial}{\partial x_i}$ its associated vector field.

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