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Cofactors and equilibria for polynomial vector fields

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We present a relationship between the existence of equilibrium points of differential systems and the cofactors of the invariant algebraic curves and the exponential factors of the system.

1. Introduction and statement of the main results

A *complex planar polynomial differential system of degree d* is a differential system of the form

$$\dot{x} = P(x,y), \qquad \dot{y} = Q(x,y), \tag{1.1}$$

where $P, Q \in \mathbb{C}[x,y]$ are coprime and d is the maximum of the degrees of the polynomials P and Q. As usual, $\mathbb{C}[x,y]$ denotes the ring of all polynomials in the variables x and y with coefficients in the set of complex numbers $\mathbb{C}$. The vector field associated with (1.1) is

$$\mathcal{X}(x,y) = P(x,y)\frac{\partial}{\partial x} + Q(x,y)\frac{\partial}{\partial y}.$$

Of course, we say that $\mathcal{X}$ is a *polynomial vector field of degree d*. In what follows we make no distinction between the polynomial differential system (1.1) and its vector field $\mathcal{X}$.

Let U be an open and dense subset of $\mathbb{C}^2$. A *first integral* of X in U is a locally non-constant analytic function $H\colon U \to \mathbb{C}$, possibly multi-valued, that is constant on all the solutions of X contained in U, i.e. $\mathcal{X}H = 0$ in the points of U. In this case we also say that X is *integrable* on U.

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