CONVEX CENTRAL CONFIGURATIONS OF THE 4–BODY PROBLEM WITH TWO PAIRS OF EQUAL ADJACENT MASSES

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ABSTRACT. We study the convex central configurations of the 4–body problem assuming that they have two pairs of equal masses located at two adjacent vertices of a convex quadrilateral. Under these assumptions we prove that the isosceles trapezoid is the unique central configuration.

1. Introduction and statement of the main results

The classical Newtonian n–body problem studies a system formed by n punctual bodies with positives masses $m_1, \dots, m_n$ and position vectors $r_1, \dots, r_n$ in $\mathbb{R}^d$, $d = 2, 3$, interacting under the Newton's gravitational law [20]. The equations of motion of this problem are

$$\ddot{r}_i = \frac{d^2 r_i}{dt^2} = -\sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{m_j}{r_{ij}^3} (r_i - r_j), \tag{1}$$

for $i = 1, \dots, n$, where $r_{ij} = |r_i - r_j|$ is the Euclidean distance between the bodies at r_i and r_j, and t is the independent variable called time. Taking the unit of mass conveniently we can assume that the gravitational constant $G = 1$ in (1).

An interesting class of particular solutions of the n–body problem (1) are the *homographic* solutions in which the shape of the configuration is preserved as time varies. The first homographic solutions were found by Euler [10] and Lagrange [13] in the 3–body problem.

We say that at a given instant $t = t_0$ the n bodies are in a *central configuration* if for all $i = 1, \dots, n$ there exists a constant $\lambda \neq 0$ such that $\ddot{r}_i = \lambda(r_i - c)$ where c is the center of mass of the n bodies, that is

$$c = \frac{1}{m_1 + \ldots + m_n} \sum_{j=1}^{n} m_j \, r_j.$$

Such configurations are closely related with homographic solutions. In fact, the configuration of bodies at any time in a homographic solution is a central configuration. For more details see for instance [19, 22, 23, 25].

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