# ANALYTIC INTEGRABILITY OF THE BIANCHI CLASS A COSMOLOGICAL MODELS WITH $0 \leq k < 1$

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ABSTRACT. There are many works studying the integrability of the Bianchi class A cosmologies with $k = 1$. Here we characterize the analytic integrability of the Bianchi class A cosmological models when $0 \leq k < 1$.

## 1. INTRODUCTION

Bianchi models describe space-times which are foliated by homogeneous (and so we have three dimensional Lie algebras) hypersurfaces of constant time. Bianchi [2, 3] was the first to classify three dimensional Lie algebras which are nonisomorphic. There are nine types of models according to the dimension $n$ of the algebra:

(a) $n = 0$: type I;
(b) $n = 1$: types II, III;
(c) $n = 2$: types IV, V, VI, VII;
(d) $n = 3$: types VIII, IX.

If we consider $X_1, X_2, X_3$ an appropriate basis of the 3-dimensional Lie Algebra, then the classification depends on a scalar $a \in \mathbb{R}$ and a vector $(n_1, n_2, n_3)$, with $n_i \in \{+1, -1, 0\}$ such that

$$[X_1, X_2] = n_3 X_3, \quad [X_2, X_3] = n_1 X_1 - a X_2, \quad [X_3, X_1] = n_2 X_2 + a X_1,$$

where $[,]$ is the Lie bracket. In particular for $a = 0$ we obtain models of class A and for $a \neq 0$ we obtain models of class B. A good reference for the Bianchi models is Bogoyavlensky [4].

In a cosmological model Einstein's equations connect the geometry of the space-time with the properties of the matter. The matter occupying the space-time is determined by the stress energy tensor of the matter. In our study we follow [4] and we consider the hydrodynamical tensor of the matter. We will work with an equation of state of matter of the form $p = k\varepsilon$, where $\varepsilon$ is the energy density of the matter, $p$ is the pressure and $0 \leq k \leq 1$.

Following [4] the Einstein equations for the homogenous cosmologies of class A without motion of matter can be formalized as a Hamiltonian system in the phase space $p_i, q_i$ for

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