International Journal of Bifurcation and Chaos, Vol. 23, No. 2 (2013) 1350029 (10 pages)

DOI: 10.1142/S0218127413500296

POLYNOMIAL VECTOR FIELDS IN $\mathbb{R}^3$ WITH INFINITELY MANY LIMIT CYCLES

ANTONI FERRAGUT*
Departament de Matemàtica Aplicada I,
Universitat Politècnica de Catalunya, Av. Diagonal,
647, 08028, Barcelona, Catalonia, Spain
antoni.ferragut@upc.edu

JAUME LLIBRE†
Departament de Matemàtiques,
Universitat Autònoma de Barcelona, Edifici C,
08193 Bellaterra, Barcelona, Catalonia, Spain
jllibre@mat.uab.cat

CHARA PANTAZI‡
Departament de Matemàtica Aplicada I (EPSEB),
Universitat Politècnica de Catalunya,
Av. Doctor Marañón, 44-50,
Barcelona, Catalonia, Spain
chara.pantazi@upc.edu

Received December 1, 2011; Revised July 4, 2012

We provide a constructive method to obtain polynomial vector fields in $\mathbb{R}^3$ having infinitely many limit cycles starting from polynomial vector fields in $\mathbb{R}^2$ with a period annulus. We present two examples of polynomial vector fields in $\mathbb{R}^3$ having infinitely many limit cycles, one of them of degree 2 and the other one of degree 12. The main tools of our method are the Melnikov integral and the Hamiltonian structure.

Keywords: Limit cycle; Melnikov integral; polynomial vector fields in $\mathbb{R}^3$.

1. Introduction

A vector field $X : \mathbb{R}^3 \to \mathbb{R}^3$ of the form $X = (P, Q, R)$ is called a *polynomial vector field of degree m* if P, Q and R are polynomials and m is the maximum of the degrees of P, Q and R.

A *limit cycle* of a vector field is an isolated periodic solution in the set of all periodic solutions of this vector field.

In this paper, we provide a method to construct polynomial vector fields in $\mathbb{R}^3$ having infinitely

*Author for correspondence; partially supported by grants MTM2008–03437, *Juan de la Cierva*, 2009SGR410 and MTM2009–14163–C02–02.
†Partially supported by grants MTM2008–03437 and 2009SGR410 and by ICREA Academia.
‡Partially supported by grants MTM2008–03437, MTM2009–06973 and 2009SGR859.