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Original article

Center conditions and cyclicity for a family of cubic systems: Computer algebra approach

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Abstract

Using methods of computational algebra we obtain an upper bound for the cyclicity of a family of cubic systems. To that end we overcome the problem of nonradicality of the associated Bautin ideal by moving from the ring of polynomials to a coordinate ring. Finally, we also determine the number of limit cycles bifurcating from each component of the center variety.

Keywords: Cyclicity; Limit cycles; Center-focus problem

1. Introduction

We consider systems of ordinary differential equations on $\mathbb{R}^2$ of the form

$$\begin{aligned} \dot{u} &= \lambda u - v + \sum_{j+k=2}^{N} A_{j,k} u^j v^k = P(u, v), \\ \dot{v} &= u + \lambda v + \sum_{j+k=2}^{N} B_{j,k} u^j v^k = Q(u, v), \end{aligned} \tag{1}$$

where λ is arbitrarily close to zero (possibly zero). The *degree* of system (1) is $N = \max\{\deg P, \deg Q\}$. Depending on nonlinear terms the origin of system (1) is either a *center* (every orbit is an oval surrounding the origin), or a *focus* (every trajectory spirals towards or away from the origin). The problem of distinguishing between a center and a focus is called the *center* or the *center-focus* problem, for more details, see e.g. [20].

For system (1) denote by (λ, A, B) the set of its parameters λ, $A_{j,k}$ and $B_{j,k}$, and by $E(\lambda, A, B)$ the associated space of parameters. Let also $n_{(\lambda,A,B),\varepsilon}$ denote the number of limit cycles of system (1) that lie wholly within an ε-neighborhood of the origin. We define the key concept of this article, namely the cyclicity of a singular point.

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