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On the Darboux integrability of a cubic CRN model in $\mathbb{R}^5$

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ABSTRACT

We study the Darboux integrability of a differential system in $\mathbb{R}^5$ with parameters coming from a chemical reaction model. In particular, we find all its Darboux polynomials and exponential factors and we prove that it is not Darboux integrable.

1. Introduction and statement of the main result

Consider an n-dimensional polynomial differential system of degree $d \in \mathbb{N}$

$$\dot{\mathbf{x}} = P(\mathbf{x}), \quad \mathbf{x} \in \mathbb{R}^n, \tag{1.1}$$

where $P(\mathbf{x}) = (P_1(\mathbf{x}), \dots, P_n(\mathbf{x}))$, $P_i \in \mathbb{C}[\mathbf{x}]$, and the dot denotes derivative with respect to the independent variable t.

A function $H(\mathbf{x})$ is a *first integral* of system (1.1) if it is continuous and defined in a full Lebesgue measure subset $\Omega \subseteq \mathbb{R}^n$, is not locally constant on any positive Lebesgue measure subset of Ω and moreover is constant along each orbit in Ω of system (1.1). If H is $\mathcal{C}^1$ and we name $\mathcal{X}$ the vector field associated to system (1.1), then we have

$$\mathcal{X}(H) = P_1 \frac{\partial H}{\partial x_1} + \cdots + P_n \frac{\partial H}{\partial x_n} = 0.$$

System (1.1) is $\mathcal{C}^k$*-completely integrable*in Ω if it has $n-1$ functionally independent C^k first integrals in Ω. Recall that k functions $H_1(\mathbf{x}), \dots, H_k(\mathbf{x})$ are *functionally independent* in Ω if the matrix of gradients $(\nabla H_1, \dots, \nabla H_k)$ has rank k in a full Lebesgue measure subset of Ω.

For an n-dimensional system of differential equations the existence of some first integrals reduces the complexity of its dynamics and the existence of $n-1$ functionally independent first integrals solves completely the problem (at least theoretically) of determining its phase portrait. In general for a given differential system it is a difficult problem to determine the existence or non-existence of first integrals.

During recent years the interest in the study of the integrability of differential equations has attracted much attention from the mathematical community. Darboux theory of integrability plays a central role in the integrability of the polynomial differential systems since it gives a sufficient condition for the integrability inside a wide family of functions. We highlight that it works for real or complex polynomial differential systems and that the study of complex algebraic solutions is necessary for obtaining all the real first integrals of a real polynomial differential system.

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