

On the Complete Integrability of the Raychaudhuri Differential System in $\mathbb{R}^4$ and of a CRNT Model in $\mathbb{R}^5$

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Abstract We study the Darboux integrability of two differential systems with parameters: the Raychaudhuri equation (a relativistic model in $\mathbb{R}^4$) and a chemical reaction model in $\mathbb{R}^5$. We prove that the first one is completely integrable and that the first integrals are of Darboux type. This is the first four-dimensional realistic non-trivial model which is completely integrable with first integrals of Darboux type and for which for a full Lebesgue measure set of the values of the parameters the three linearly independent first integrals are rational. For the second one, we find all its Darboux polynomials and exponential factors and we prove that it is not Darboux integrable.

Keywords Darboux polynomial · Exponential factor · Darboux integrability · Raychaudhuri equation · Chemical reaction network

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