

On Darboux integrability of Edelstein's reaction system in $\mathbb{R}^3$

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Abstract

We consider Edelstein's dynamical system of three reversible reactions in $\mathbb{R}^3$ and show that it is not Darboux integrable. To do so we characterize its polynomial first integrals, Darboux polynomials and exponential factors.

1 Introduction

We consider Edelstein's system of three reversible biochemical reactions among three molecular species [4]:



where $\alpha_1, \alpha_2, \beta_1, \beta_2, \gamma_1, \gamma_2$ are positive reaction rate constants. Under mass-action kinetics the evolution of the species concentrations is described by the following ODE system of degree 2,

$$\begin{aligned} \dot{x} &= \alpha_1 x + \beta_2 z - \alpha_2 x^2 - \beta_1 xy \\ \dot{y} &= -\gamma_2 y + (\gamma_1 + \beta_2)z - \beta_1 xy \\ \dot{z} &= \gamma_2 y - (\gamma_1 + \beta_2)z + \beta_1 xy, \end{aligned} \tag{2}$$