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Computer-assisted techniques for the verification of the Chebyshev property of Abelian integrals

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ABSTRACT

We develop techniques for the verification of the Chebyshev property of Abelian integrals. These techniques are a combination of theoretical results, analysis of asymptotic behavior of Wronskians, and rigorous computations based on interval arithmetic. We apply this approach to tackle a conjecture formulated by Dumortier and Roussarie in [F. Dumortier, R. Roussarie, Birth of canard cycles, Discrete Contin. Dyn. Syst. 2 (2009) 723–781], which we are able to prove for $q \leqslant 2$.

1. Introduction and setting of the problem

The present paper addresses the problem of verifying when a collection of Abelian integrals form an *extended complete Chebyshev system* (ECT-system for short). This problem arises in the context of the second part of *Hilbert's* 16*th problem* [16], that asks about the maximum number and location of limit cycles of a planar polynomial vector field of degree d. Solving this problem even for the case $d = 2$ seems to be out of reach at the present state of knowledge (see [19,23] for a survey of the recent results on the subject). Arnold [2] proposed a weaker version of this problem, the so-called *infinitesimal Hilbert's* 16*th problem*. Let ω be a real 1-form with polynomial coefficients of degree at most d. Consider a real polynomial H of degree $d+1$ in the plane. A closed connected component of a level curve $H = h$ is called an oval of H and denoted by γ_h. These ovals form continuous families,

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