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First derivative of the period function with applications

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Abstract

Given a centre of a planar differential system, we extend the use of the Lie bracket to the determination of the monotonicity character of the period function. As far as we know, there are no general methods to study this function, and the use of commutators and Lie bracket was restricted to prove isochronicity. We give several examples and a special method which simplifies the computations when a first integral is known.

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1. Introduction

In the latest years, there have been many developments concerning the problem of centres for systems of ordinary differential equations on the plane. By one side, improvements have been done in the direction of solving the centre-focus problem (see [14] or [24] for instance, and the references therein); however, the problem is far

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