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A characterization of isochronous centres in terms of symmetries

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Abstract

We present a description of isochronous centres of planar vector fields X by means of their groups of symmetries. More precisely, given a normalizer U of X (i.e., $[X, U] = \mu X$, where μ is a scalar function), we provide a necessary and sufficient isochronicity condition based on μ. This criterion extends the result of Sabatini and Villarini that establishes the equivalence between isochronicity and the existence of commutators ($[X, U] = 0$). We put also special emphasis on the mechanical aspects of isochronicity; this point of view forces a deeper insight into the potential and quadratic-like Hamiltonian systems. For these families we provide new ways to find isochronous centres, alternative to those already known from the literature.

1. Introduction

Along this paper we suppose to have a vector field X on the plane with a centre at some critical point p. We are mainly concerned about the *isochronicity problem*, that is, to determine whether the periodic orbits around p have the same period or not.

The pioneering works wondering about isochronicity were already given in the sixties by Levin and Shatz, Loud, Pleshkan and Urabe (see [7], [8], [11] and [14]). Nevertheless, until the beginning of this decade, the problem has not been deeply considered. The most relevant works are those of Sabatini and Villarini (see [13] and [15] and the references therein), where they settled the strong relationship between Lie brackets and isochronicity. In particular, they proved that p is an isochronous centre if and only if there

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