# LIMIT CYCLES OF CONTINUOUS AND DISCONTINUOUS PIECEWISE–LINEAR DIFFERENTIAL SYSTEMS IN $\mathbb{R}^3$

BRUNO R. DE FREITAS[1], JAUME LLIBRE[2] AND JOAO C. MEDRADO[3]

Abstract. We study the limit cycles of two families of piecewise–linear differential systems in $\mathbb{R}^3$ with two pieces separated by a plane $\Sigma$. In one family the differential systems are only continuous on the plane $\Sigma$, and in the other family they are only discontinuous on the plane $\Sigma$.

The usual tool for studying these limit cycles is the Poincaré map, but here we shall use recent results which extend the averaging theory to continuous and discontinuous differential systems.

All the computations have been checked with the algebraic manipulator mathematica.

## 1. Introduction and statement of the main results

The study of piecewise linear differential systems essentially started with Andronov, Vitt and Khaikin [1] and still continues to receive attention by researchers. The continuous and discontinuous piecewise–linear differential systems plays an important role inside the nonlinear dynamical systems. First they appear in a natural way in nonlinear engineering models, where certain devices are accurately modeled by such differential systems, see for instance the books of di Bernardo, Budd, Champneys and Kowalczyk [3], and Simpson [28], and the survey of Makarenkov and Lamb [26], and the hundreds of references quoted in these last three works. Moreover these kind of differential systems are frequent in applications from electronic engineering and nonlinear control systems, where they cannot be considered as idealized models; they are also used in mathematical biology as well, see for instance [7, 29, 30, 31].

There are many studies of the limit cycles of continuous and discontinuous piecewise–linear differential systems in $\mathbb{R}^2$ with two pieces separated by a straight line, see for instance [2, 4, 6, 9, 10, 11, 12, 13, 14, 15, 16, 21, 22, 23, 25, 27]. But there are few results about the limit cycles of continuous and discontinuous piecewise–linear differential systems in $\mathbb{R}^3$ with two pieces separated by a plane. The objective of this work is to study the limit cycles of some of these last systems.

2010 *Mathematics Subject Classification.* Primary 34C05, 34C07, 37G15.

*Key words and phrases.* non–smooth differential system, limit cycle, piecewise linear differential system.