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A theoretical basis for the Harmonic Balance Method

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ABSTRACT

The Harmonic Balance Method provides a heuristic approach for finding truncated Fourier series as an approximation to the periodic solutions of ordinary differential equations. Another natural way for obtaining these types of approximations consists in applying numerical methods. In this paper we recover the pioneering results of Stokes and Urabe that provide a theoretical basis for proving that near these truncated series, whatever is the way they have been obtained, there are actual periodic solutions of the equation. We will restrict our attention to one-dimensional non-autonomous ordinary differential equations, and we apply the obtained results to a concrete example coming from a rigid cubic system.

1. Introduction and main results

Consider the real non-autonomous differential equation

$$x' = X(x,t), \tag{1}$$

where the prime denotes the derivative with respect to t, $X : \Omega \times [0, 2\pi] \to \mathbb{R}$ is a C^2-function, 2π-periodic in t, and $\Omega \subset \mathbb{R}$ is a given open interval.

There are several methods for finding approximations to the periodic solutions of (1). For instance, the Harmonic Balance Method (HBM), recalled in Section 2.1, or simply the numerical approximations of the solutions of the differential equations. In any case, from all the methods we can get a truncated

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