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Geometrical conditions for the stability of orbits in planar systems

BY R. A. GARCIA

Departamento de Matemática, Universidade Federal de Goiás Cx. Postal 131, *Goiânia, Brazil*

A. GASULL[1] AND A. GUILLAMON[1,2]

Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 *Bellaterra, Catalonia, Spain*
e-mail address: GASULL@MAT.UAB.ES.

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Abstract

Given a vector field X on the real plane, we study the influence of the curvature of the orbits of $\dot{x} = X^{\perp}(x)$ in the stability of those of the system $\dot{x} = X(x)$. We pay special attention to the case in which this curvature is negative in the whole plane. Under this assumption, we classify the possible critical points and give a criterion for a point to be globally asymptotically stable. In the general case, we also provide expressions for the first three derivatives of the Poincaré map associated to a periodic orbit in terms of geometrical quantities.

1. *Introduction and main results*

It is known that the divergence, div X, of a C^1 vector field $X = (P, Q)$ plays an important role in many problems related to systems of ordinary differential equations on the plane

$$\frac{d\mathrm{x}}{dt} = X(\mathrm{x}) = (P(\mathrm{x}), Q(\mathrm{x})), \quad \mathrm{x} = (x, y) \in \mathbb{R}^2, \tag{1}$$

such as the classification of critical points, the non-existence of periodic orbits, the stability of limit cycles, the stability of graphics or the global stability of a critical point, for instance.

On the other hand, one may think that the function div X is not the only one that can control this kind of fact. Roughly speaking, what one intends to control in those problems is the tendency of the trajectories of the flow of (1) to join among them or to separate. However, dealing with the divergence, one does not speculate only about the geometry of the orbits, but also about their parametrization. In some problems, it may happen that the time is not an important factor and the behaviour of the orbits depends only on their geometry. Then, hypotheses on the divergence would turn out to be too strong. Our aim is to get a kind of substitute for the divergence, some operator that can also decide about the behaviour of the flow of (1).

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