

On the period function for a family of complex differential equations

Antonio Garijo^{a,*}, Armengol Gasull^b, Xavier Jarque^c

^a*Dep. d'Eng. Informàtica i Matemàtiques, Universitat Rovira i Virgili, Av. Països Catalans, 26, 43007 Tarragona, Spain*

^b*Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain*

^c*Departament de Matemàtica Aplicada i Anàlisi, Universitat de Barcelona, Gran Via, 585, 08007 Barcelona, Spain*

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Abstract

We consider planar differential equations of the form $\dot{z} = f(z)g(\bar{z})$ being $f(z)$ and $g(z)$ holomorphic functions and prove that if $g(z)$ is not constant then for any continuum of period orbits the period function has at most one isolated critical period, which is a minimum. Among other implications, the paper extends a well-known result for meromorphic equations, $\dot{z} = h(z)$, that says that any continuum of periodic orbits has a constant period function.

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1. Introduction

Analytic planar autonomous differential equations can be written in complex coordinates as

$$\dot{z} = h(z, \bar{z}), \quad z \in \mathbb{C},$$

* Corresponding author.

E-mail addresses: antonio.garijo@urv.net (A. Garijo), gasull@mat.uab.es (A. Gasull), xavier.jarque@ub.edu (X. Jarque).