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A note on the period function for certain planar vector fields

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Dedicated to Robert Devaney on the occasion of his 60th birthday

Consider a smooth planar autonomous differential equation having a period annulus, $\mathcal{P}$. We present a new criterion to ensure that the period function has at most one critical period on $\mathcal{P}$. Our result has a compact form when the differential equation is written as $\dot{z} = F(z, \bar{z})$. It is based on a suitable representation formula of the derivative of the period function which uses the infinitesimal generator associated to the continua of periodic orbits. We apply the criterion to several particular cases of the equation $\dot{z} = f(z)g(\bar{z})/h(z, \bar{z})$, where $f(z)$ and $g(\bar{z})$ are holomorphic functions and h is a C^2 smooth real valuated function.

Keywords: meromorphic vector fields; periodic orbit; period function; critical period

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1. Introduction

In recent years, there has been an increasing interest in studying the period function for planar smooth vector fields with a period annulus $\mathcal{P}$, mainly associated to a centre. Interest has focussed on the existence and characterization of isochronous centres (constant period function; see [2,4–6,13,19,22,23]), monotonicity of the period function (see [2,24,25,28]), existence and uniqueness of critical periods (see [3,9,11,16,29]), etc. Also the bifurcation diagram of the period function for several families of planar polynomial vector fields has been studied in several works (see [3,17,18,20]).

In this paper, which is a continuation of [9], we are interested in giving a new criterion of non existence or uniqueness of critical periods for continua of periodic orbits. We stress that our result is not restricted to period annuli associated to a centre. As we will see in Subsection 3.1, sometimes, in this later case, an alternative approach can also be considered.

Any smooth planar autonomous differential equations can be written in complex coordinates as

$$\dot{z} = F(z, \bar{z}), \quad z \in \mathbb{C}, \tag{1}$$

where the dot indicates the derivative with respect the real variable t.

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