



Note

Analytic nilpotent centers as limits of nondegenerate centers revisited

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ARTICLE INFO

Article history:

Received 17 March 2015

Available online 20 April 2016

Submitted by Y. Huang

Keywords:

Nilpotent center

Poincaré–Liapunov constants

ABSTRACT

We prove that all the nilpotent centers of planar analytic differential systems are limit of centers with purely imaginary eigenvalues, and consequently the Poincaré–Liapunov method to detect centers with purely imaginary eigenvalues can be used to detect nilpotent centers.

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1. Introduction and statement of the main result

Consider the analytic family

$$\dot{x} = y + F_1(x, y; \lambda), \quad \dot{y} = F_2(x, y; \lambda) \quad (1)$$

with parameters $\lambda \in \mathbb{R}^p$ and having a nilpotent singularity at the origin.

In the papers [8–10] is stated a theorem which is slightly modified each time in order to correct the previous version but surprisingly it has never been properly written. Anyway we want to emphasize that the ideas presented in [8] have merit and in our opinion they are new and useful for understanding the nilpotent center problem mainly due to the computational algorithm that is derived from it. Here we present the correct statement and the right proof. We will present one counterexample for showing that the previous version of the mentioned theorem does not work. Also we end with an example for showing the analysis of a nilpotent center problem on a family using the right method provided here.

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