PERGAMON

Applied Mathematics Letters 16 (2003) 1137–1141

Applied Mathematics Letters

www.elsevier.com/locate/aml

Generalized Cofactors and Nonlinear Superposition Principles

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(Received November 2001; accepted May 2002)

Abstract—It is known from Lie's works that the only ordinary differential equation of first order in which the knowledge of a certain number of particular solutions allows the construction of a fundamental set of solutions is, excepting changes of variables, the Riccati equation. For planar complex polynomial differential systems, the classical Darboux integrability theory exists based on the fact that a sufficient number of invariant algebraic curves permits the construction of a first integral or an inverse integrating factor. In this paper, we present a generalization of the Darboux integrability theory based on the definition of generalized cofactors.

Keywords—Nonlinear differential equations, Nonlinear superposition, Trascendental solutions, Non-Liouvillian first integral.

1. INTRODUCTION

By definition, a *complex* (respectively, *real*) *planar polynomial differential system* or simply a *polynomial system* will be a differential system of the form

$$\frac{dx}{dt} = \dot{x} = P(x,y), \qquad \frac{dy}{dt} = \dot{y} = Q(x,y), \tag{1}$$

in which $P, Q \in \mathbb{C}[x,y]$ (respectively, $\mathbb{R}[x,y]$) are polynomials in the complex (respectively, real) variables x and y and the independent one (the time) t is real. Throughout this paper, we will denote by $m = \max\{\deg P, \deg Q\}$ the *degree* of system (1). Obviously, we can also express system (1) as the differential equation

$$\frac{dy}{dx} = \frac{Q(x,y)}{P(x,y)}, \tag{2}$$

and, moreover, we also associate to system (1) the vector field $\mathcal{X}$ defined by $\mathcal{X} = P(x,y)\frac{\partial}{\partial x} + Q(x,y)\frac{\partial}{\partial y}$.

The authors are partially supported by MCYT Grant No. BFM2002-04236-C02-01. The second author is partially supported by DURSI of Government of Catalonia's Acció Integrada ACI2002-24.
We would like to thank Prof. H. Giacomini for many valuable comments and suggestions.

0893-9659/03/$ - see front matter Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TeX
doi: 10.1016/S0893-9659(03)00156-3