Available online at www.sciencedirect.com
ScienceDirect
Journal of Symbolic Computation 41 (2006) 943–958
Journal of Symbolic Computation
www.elsevier.com/locate/jsc

A necessary condition in the monodromy problem for analytic differential equations on the plane

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Received 4 February 2005; accepted 24 April 2006
Available online 23 June 2006

Abstract

In this paper we give a very easy to compute necessary condition in the monodromy problem for all singular point of analytic differential systems in the real plane. Our main tool is considering the analytic function, *angular speed*, and studying its limit through straight lines to the singular point.

Keywords: Planar vector field; Monodromy problem; Characteristic orbit

1. Introduction and statement of the main results

We consider two-dimensional autonomous systems of real differential equations of the form

$$\dot{x} = P(x, y), \quad \dot{y} = Q(x, y), \tag{1}$$

where $\dot{} = \frac{\mathrm{d}}{\mathrm{d}t}$ and $P(x, y)$ and $Q(x, y)$ are analytic functions defined in a neighborhood of the origin such that $P(0, 0) = Q(0, 0) = 0$ and there is no $d(x, y)$, non-unit element of the ring of analytic functions defined in a neighborhood of the origin, which divides both $P(x, y)$ and $Q(x, y)$.

Definition 1. A point $(x_0, y_0) \in \mathbb{R}^2$ is a *singular* point for system (1) if both $P(x_0, y_0) = 0$ and $Q(x_0, y_0) = 0$.

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0747-7171/$ - see front matter
doi:10.1016/j.jsc.2006.04.007