

On McMullen-like mappings

Antonio Garijo¹ and Sébastien Godillon

Abstract. We introduce a generalization of particular dynamical behavior for rational maps. In 1988, C. McMullen showed that the Julia set of $f_\lambda(z) = z^n + \lambda/z^d$ for $|\lambda| \neq 0$ small enough is a Cantor set of circles if and only if $1/n + 1/d < 1$ holds. Several other specific singular perturbations of polynomials have been studied in recent years, all have parameter values where a Cantor set of circles is present in the associated Julia set. We unify these examples by defining a McMullen-like mapping as a rational map f associated to a hyperbolic postcritically finite polynomial P and a pole data \mathcal{D} where we encode the location of every pole of f and the local degree at each pole. As for the McMullen family f_λ , we characterize a McMullen-like mapping using an arithmetic condition depending only on (P, \mathcal{D}) . We show how to check the definition in practice providing new explicit examples of McMullen-like mappings for which a complete topological description of their Julia sets is made.

Mathematics Subject Classification (2010). 37F10; 37F20.

Keywords. Complex dynamics, Julia sets, rational maps, McMullen family.

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¹ The first author is partially supported by the Catalan grant 2009SGR-792 and by the Spanish grant MTM-2008-01486 Consolider (including a FEDER contribution).