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Global dynamics of the real secant method*

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Abstract

We investigate the root finding algorithm given by the secant method applied to a real polynomial p as a discrete dynamical system defined on $\mathbb{R}^2$. We study the shape and distribution of the basins of attraction associated to the roots of p, and we also show the existence of other stable dynamics that might affect the efficiency of the algorithm. Finally we extend the secant map to the punctured torus $\mathbb{T}^2_\infty$ which allow us to better understand the dynamics of the secant method near ∞ and facilitate the use of the secant map as a method to find all roots of a polynomial.

Keywords: root finding algorithms, rational iteration on the plane, dynamical systems, secant method
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(Some figures may appear in colour only in the online journal)

1. Introduction

Root finding algorithms

$$x_{n+1} = R\left(x_{n-\ell}, \ldots, x_n\right), \ \ell \geqslant 0, n \geqslant \ell, \ x \in X \tag{1}$$

are iterative systems so that for most initial *seeds* $(x_0, \ldots, x_\ell)$ the sequence $\{x_n\}_{n\geqslant 0}$ converges to a solution of a given nonlinear equation, namely $F(x) = 0, \ x \in X$. Since many real problems can be modelled in terms of nonlinear equations which do not admit explicit solutions,

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