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JOINING POLYNOMIAL AND EXPONENTIAL COMBINATORICS FOR SOME ENTIRE MAPS

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Abstract

We consider families of entire transcendental maps given by $F_{\lambda,m}(z) = \lambda z^m \exp(z)$ where $m \geq 2$. All these maps have a super-attracting fixed point at $z = 0$ and a free critical point at $z = -m$. In parameter planes we focus on the capture zones, i.e., we consider λ values for which the free critical point belongs to the basin of attraction of $z = 0$. We explain the connection between the dynamics near zero and the dynamics near infinity at the boundary of the immediate basin of attraction of the origin, thus, joining together exponential and polynomial behaviors in the same dynamical plane.

1. Introduction

In this paper, we combine symbolic dynamics with polynomial-like theory to investigate the combinatorics of the Julia set of the families of transcendental entire functions

$$F_{\lambda,m}(z) = \lambda z^m e^z, \tag{1.1}$$

with $m \geq 2$ and $\lambda \in \mathbb{C} \setminus \{0\}$. The Julia set of an entire map f, $J(f)$, is the set of points where the family of iterates $\{f^n\}$ fails to be a normal family. Its complement in $\mathbb{C}$ is an open set of the plane known as Fatou set, where the dynamics is tame.

For all functions in (1.1), 0 is a critical and asymptotic value and $F_{\lambda,m}(-m) = \lambda(-m)^m \exp(-m)$ is a critical value. Therefore, these maps belong to a general class of entire transcendental maps with only finitely many critical and asymptotic values also known as *critically finite*. The interest on critically finite maps resides in that they resemble rational maps, as their Fatou set contains neither wandering nor Baker domains (see [**2**], [**14**], [**15**] and [**17**]). In contrast, the point at infinity plays a

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