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Non-landing hairs in Sierpiński curve Julia sets of transcendental entire maps

by

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Abstract. We consider the family of transcendental entire maps given by $f_a(z) = a(z-(1-a))\exp(z+a)$ where a is a complex parameter. Every map has a superattracting fixed point at $z=-a$ and an asymptotic value at $z=0$. For $a>1$ the Julia set of f_a is known to be homeomorphic to the Sierpiński universal curve, thus containing embedded copies of any one-dimensional plane continuum. In this paper we study subcontinua of the Julia set that can be defined in a combinatorial manner. In particular, we show the existence of non-landing hairs with prescribed combinatorics embedded in the Julia set for all parameters $a \geq 3$. We also study the relation between non-landing hairs and the immediate basin of attraction of $z=-a$. Even though each non-landing hair accumulates on the boundary of the immediate basin at a single point, its closure is an indecomposable subcontinuum of the Julia set.

1. Introduction. Let $f:\mathbb{C}\to\mathbb{C}$ be a transcendental entire map. The *Fatou set* $\mathcal{F}(f)$ is the largest open set where iterates of f form a normal family. Its complement in $\mathbb{C}$ is the *Julia set* $\mathcal{J}(f)$ and it is a non-empty unbounded subset of the plane. When the set of singular values is bounded, we say f is of *bounded singular type* and denote this class of maps by $\mathcal{B}$. It has been shown in [Ba] and [R1] that the Julia set of a hyperbolic map in $\mathcal{B}$ contains uncountably many unbounded curves, usually known as *hairs* [DT]. A hair is said to *land* if it is homeomorphic to the half-closed ray $[0,+\infty)$. The point corresponding to $t=0$ is known as the *endpoint* of the hair. In contrast, if its accumulation set is a non-trivial continuum, we obtain a *non-landing* hair.

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