CLASSICAL PLANAR ALGEBRAIC CURVES REALIZABLE BY QUADRATIC POLYNOMIAL DIFFERENTIAL SYSTEMS

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Abstract. In this paper we show examples of planar quadratic differential systems having some famous planar invariant algebraic curves. We carry out a non exhaustive classification taking into account the degree of the invariant algebraic curve. Also we pay particular attention to the Darboux integrability of the systems.

1. Introduction

Throughout this work we will consider quadratic polynomial differential systems

$$\dot{x} = P(x, y) \ , \qquad \dot{y} = Q(x, y) \ , \tag{1}$$

defined on $\mathbb{R}^2$ or *quadratic systems* for short. This name comes from the restriction that $P, Q \in \mathbb{R}[x, y]$ are polynomials with real coefficients such that $2 = \max\{\deg P, \deg Q\}$. Here the dot denotes, as usual, differentiation with respect to the time t. We also denote $\mathcal{X} = P(x, y)\partial_x + Q(x, y)\partial_y$ the vector field associated to system (1).

Quadratic systems appear very often in several branches of science, as biology, physics, chemistry, mechanics, etc. See for instance [1, 2] for a summary of several properties of these systems.

From the mathematical point of view quadratic systems are perhaps the most simple nonlinear differential systems. Despite its simplicity there are important open questions around them. May be the most important open problem involving quadratic systems is the famous *Hilbert 16th problem* restricted to them. The problem was posed by David Hilbert at the Paris conference of the International Congress of Mathematicians in 1900, together with the other 22 problems. Actually the problem consists of two parts: (i) an investigation of the relative positions of the branches of real algebraic curves of fixed degree; (ii) the determination of the upper bound for the

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2010 *Mathematics Subject Classification.* 34C05, 34C08.

Key words and phrases. Quadratic differential system, invariant algebraic curve, Darboux integrability.

The first author is partially supported by a MINECO grant number MTM2014-53703-P and an AGAUR grant number 2014SGR 1204. The second author is partially supported by a MINECO grants MTM2016-77278-P and MTM2013-40998-P, an AGAUR grant number 2014SGR 568, and a FP7-PEOPLE-2012-IRSES number 318999.