EXISTENCE OF PERIODIC SOLUTIONS FOR A CLASS OF SECOND ORDER ORDINARY DIFFERENTIAL EQUATIONS

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ABSTRACT. We provide sufficient conditions for the existence of a periodic solution for a class of second order differential equations of the form $\ddot{x} + g(x) = \varepsilon f(t, x, \dot{x}, \varepsilon)$, where ε is a small parameter.

1. Introduction and statement of the results

The second order differential equations of the form

$$\ddot{x} + g(x) = \varepsilon f(t, x, \dot{x}, \varepsilon),$$

have been studied by many authors because they have many applications, see for instance [2, 3, 7, 8, 9, 12, 15]. Two of the main families studied are the Duffing equations see [5, 6], ... or the forced pendulum see the nice survey [11] and the references quoted therein.

The aim of this work is to study periodic solutions of the second order differential equation

$$\ddot{x} + g(x) = \mu^{2n+1} p(t) + \mu^{4n+1} q(t, x, y, \mu), \tag{1}$$

where n is a positive integer, μ is a small parameter, and the functions

$$g(x) = x + x^{2n+1} \left(b + x h(x)\right),$$

and $h(x)$ are smooth, $b \neq 0$, $p(t)$ and $q(t, x, y, \mu)$ are smooth and periodic with period 2π in the variable t.

Let $\Gamma(x)$ the Gamma function, see for more details [1], and let α and β the first Fourier coefficients of the periodic function $p(t)$, i.e.

$$\alpha = \frac{1}{\pi} \int_0^{2\pi} p(t) \cos t \, dt, \qquad \beta = \frac{1}{\pi} \int_0^{2\pi} p(t) \sin t \, dt.$$

Then our main result is the following.

2010 *Mathematics Subject Classification.* 37G15, 37C80, 37C30.

Key words and phrases. periodic orbit, second–order differential equation, averaging theory.