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The Hopf cyclicity of the centers of a class of quintic polynomial vector fields

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Abstract

We consider families of planar polynomial vector fields having a singularity with purely imaginary eigenvalues for which a basis of its Bautin ideal $\mathcal{B}$ is known. We provide an algorithm for computing an upper bound of the Hopf cyclicity less than or equal to the Bautin depth of $\mathcal{B}$. We also present a method for studying the cyclicity problem for the Hamiltonian and the time-reversible centers without the necessity of solving previously the Dulac complex center problem associated to the larger complexified family. As application we analyze the Hopf cyclicity of the quintic polynomial family written in complex notation as $\dot{z} = iz + z\bar{z}(Az^3 + Bz^2\bar{z} + Cz\bar{z}^2 + D\bar{z}^3)$.

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