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Center cyclicity of a family of quartic polynomial differential system

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Abstract. In this paper we study the cyclicity of the centers of the quartic polynomial family written in complex notation as

$$\dot{z} = iz + z\bar{z}\big(Az^2 + Bz\bar{z} + C\bar{z}^2\big),$$

where $A, B, C \in \mathbb{C}$. We give an upper bound for the cyclicity of any non-linear center at the origin when we perturb it inside this family. Moreover we prove that this upper bound is sharp.

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1. Introduction and statement of the main results

We consider a family of planar polynomial differential systems of the form

$$\begin{aligned} \dot{x} &= \alpha x - y + P(x, y, \lambda), \\ \dot{y} &= x + \alpha y + Q(x, y, \lambda), \end{aligned} \tag{1}$$

where $P, Q \in \mathbb{R}[x, y, \lambda]$ are the polynomial nonlinearities of system (1) and $\alpha \in \mathbb{R}$, $\lambda \in \mathbb{R}^n$ are the parameters of the family. One of the main problems in the qualitative theory of real planar polynomial systems consists in distinguishing if the singular point located at the origin O of system (1) is either a *center* (i.e., it has a neighborhood U such that $U \setminus \{O\}$ is filled with periodic orbits) or a *focus* (i.e., it has a neighborhood U where all the orbits in $U \setminus \{O\}$ spiral in forward or in backward time to the origin), see [1]. Clearly, the origin of family (1) is a focus when $\alpha \neq 0$.

A characterization of system (1) having a center at the origin is given by the existence of a formal first integral (which in fact it is analytic) $H(x, y) = x^2 + y^2 + \dots$ (here the dots denote higher order terms) with $\alpha = 0$, see Poincaré [14] and Liapunov [11]. More precisely we seek for a formal series $H(x, y; \lambda) = x^2 + y^2 + \cdots$ in such a way that $\mathcal{X}_\lambda(H) = \sum_{j \geq 1} \eta_j(\lambda)(x^2 + y^2)^j$ where $\mathcal{X}_\lambda =$