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Cyclicity of a simple focus via the vanishing multiplicity of inverse integrating factors ☆

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ABSTRACT

First we provide new properties about the vanishing multiplicity of the inverse integrating factor of a planar analytic differential system at a focus. After we use this vanishing multiplicity for studying the cyclicity of foci with pure imaginary eigenvalues and with homogeneous nonlinearities of arbitrary degree having either its radial or angular speed independent of the angle variable in polar coordinates. After we study the cyclicity of a class of nilpotent foci in their analytic normal form.

1. Introduction and statement of the results

We consider planar differential systems

$$\dot{x} = P(x, y), \qquad \dot{y} = Q(x, y), \tag{1}$$

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