

On the polynomial differential systems having polynomial first integrals

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Abstract

We consider the class of complex planar polynomial differential systems having a polynomial first integral. Inside this class the systems having minimal polynomial first integrals without critical remarkable values are the Hamiltonian ones. Here we mainly study the subclass of polynomial differential systems such that their minimal polynomial first integrals have a unique critical remarkable value.

In particular we characterize all the Liénard polynomial differential systems $\dot{x} = y$, $\dot{y} = -f(x)y - g(x)$, with $f(x)$ and $g(x)$ complex polynomials in the variable x , having a minimal polynomial first integral with a unique critical remarkable value.

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1. Introduction and statement of the main results

The nonlinear ordinary differential equations or simple the differential systems appear in many branches of applied mathematics, physics, and in general in applied sciences. In general the differential systems cannot be solved explicitly, so the qualitative information provided by the theory of dynamical systems is the best that one can expect to obtain.

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