J. Differential Equations 255 (2013) 3185–3204

Contents lists available at SciVerse ScienceDirect

Journal of Differential Equations

www.elsevier.com/locate/jde

Planar quasi-homogeneous polynomial differential systems and their integrability

Belén García [a], Jaume Llibre [b], Jesús S. Pérez del Río [a,*]

[a] Departamento de Matemáticas, Universidad de Oviedo, Avda Calvo Sotelo, s/n, 33007, Oviedo, Spain
[b] Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain

ARTICLE INFO

Article history:
Received 30 July 2012
Revised 10 April 2013
Available online 7 August 2013

MSC:
37J35
34A05
34C14

Keywords:
Polynomial first integral
Analytic first integral
Quasi-homogeneous polynomial differential system
Quasi-homogeneous polynomial vector field

ABSTRACT

In this paper we study the quasi-homogeneous polynomial differential systems and provide an algorithm for obtaining all these systems with a given degree. Using this algorithm we obtain all quasi-homogeneous vector fields of degree 2 and 3.

The quasi-homogeneous polynomial differential systems are Liouvillian integrable. In particular, we characterize all the quasi-homogeneous vector fields of degree 2 and 3 having a polynomial, rational or global analytical first integral.

1. Introduction

We deal with polynomial differential systems of the form

$$\dot{x} = P(x, y), \qquad \dot{y} = Q(x, y), \tag{1}$$

with $P, Q \in \mathbb{C}[x, y]$. As usual $\mathbb{C}$ denotes the set of complex numbers and $\mathbb{C}[x, y]$ denotes the ring of all polynomials in the variables x and y with coefficients in $\mathbb{C}$. The dot denotes derivative with respect to an independent variable t, which can be real or complex. We denote by $X = (P, Q)$ the polynomial vector field associated to system (1) and we say that the degree of the system or of the vector field is $n = \max\{\deg(P), \deg(Q)\}$.

* Corresponding author.
E-mail addresses: belen.garcia@uniovi.es (B. García), jllibre@mat.uab.cat (J. Llibre), jspr@uniovi.es (J.S. Pérez del Río).

0022-0396/$ – see front matter
http://dx.doi.org/10.1016/j.jde.2013.07.032