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Limit cycles of generalized Liénard polynomial differential systems via averaging theory

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ABSTRACT

Using the averaging theory of first and second order we study the maximum number of limit cycles of the polynomial differential systems

$$\dot{x} = y, \quad \dot{y} = -x - \varepsilon(h_1(x) + p_1(x)y + q_1(x)y^2) - \varepsilon^2(h_2(x) + p_2(x)y + q_2(x)y^2),$$

which bifurcate from the periodic orbits of the linear center $\dot{x} = y,\ \dot{y} = -x$, where ε is a small parameter. If the degrees of the polynomials h_1, h_2, p_1, p_2, q_1 and q_2 are equal to n, then we prove that this maximum number is $[n/2]$ using the averaging theory of first order, where $[\cdot]$ denotes the integer part function; and this maximum number is at most n using the averaging theory of second order.

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1. Introduction and statement of the results

The second part of the 16th Hilbert's problem asks for finding an upper bound to the maximum number of limit cycles which can have the class of all planar polynomial differential systems with a fixed degree. Since this problem up to now is untractable Smale in [34] proposed to restrict it to the class of classical Liénard differential systems of the form

$$\dot{x} = y, \quad \dot{y} = -x - f(x)y, \tag{1}$$

where $f(x)$ is a polynomial, or equivalently of the form

$$\dot{x} = y - F(x), \quad \dot{y} = -x, \quad \text{where} \quad F(x) = \int f(x)\,dx.$$

For these systems in 1977 Lins, de Melo and Pugh [19] stated the conjecture that if $f(x)$ has degree $n \geqslant 1$ then system (1) has at most $[n/2]$ limit cycles. They prove this conjecture for $n = 1, 2$. The conjecture for $n = 3$ has been proved recently by Chengzi Li and Llibre in [20]. For $n \geqslant 5$ the conjecture is not true, see De Maesschalck and Dumortier [7] and Dumortier, Panazzolo and Roussarie [8]. So it remains to know if the conjecture is true or not for $n = 4$.

Many of the results on the limit cycles of polynomial differential systems have been obtained by considering limit cycles which bifurcate from a single degenerate singular point (i.e. from a Hopf bifurcation), that are called *small amplitude limit cycles*, see for instance Lloyd [27]. There are partial results concerning the number of small amplitude limit cycles for Liénard polynomial differential systems.

Another often used way for obtaining results on the limit cycles of polynomial differential systems is perturbing the linear center $\dot{x} = y, \dot{y} = -x$ inside the class of polynomial differential systems, or inside the class of classical polynomial Liénard differential systems, i.e.

$$\dot{x} = y, \quad \dot{y} = -x - \varepsilon f(x)y,$$

where ε is a small parameter. The limit cycles obtained this way are sometimes called *medium amplitude limit cycles*. Of course, the number of small or medium amplitude limit

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