RENDICONTI DEL CIRCOLO MATEMATICO DI PALERMO
Serie II, Tomo LV (2006), pp. 420-440

PHASE PORTRAITS OF THE QUADRATIC VECTOR FIELDS WITH A POLYNOMIAL FIRST INTEGRAL

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In this work we classify the phase portraits of all quadratic polynomial differential systems having a polynomial first integral.

If $H(x, y)$ is a polynomial of degree $n + 1$ then the differential system $x' = -\dfrac{\partial H}{\partial y}$, $y' = \dfrac{\partial H}{\partial x}$ is called a Hamiltonian system of degree n. We also prove that all the phase portraits that we obtain in this paper are realizable by Hamiltonian systems of degree 2.

Since we observe that all the phase portraits of the linear polynomial differential systems having a polynomial first integral are also realizable by Hamiltonian systems of degree 1, an open question appears: Are all the phase portraits of polynomial differential systems of degree n having a polynomial first integral realizable by Hamiltonian systems of degree n?

1. Introduction and main results.

Let $\mathbb{R}[x, y]$ be the ring of the polynomials in the variables x and y with coefficients in $\mathbb{R}$. In this work we consider *quadratic polynomial differential systems* in $\mathbb{R}^2$, that is,

$$(1) \qquad \frac{dx}{dt} = x' = P(x, y), \qquad \frac{dy}{dt} = y' = Q(x, y),$$

where $P, Q \in \mathbb{R}[x, y]$ and $\max\{\deg P, \deg Q\} = 2$. In the following we denote these systems simply as *quadratic systems* and we denote by $X =$

1991 *Mathematics Subject Classification.* Primary 34C05, 58F14.

Key words and phrases. Polynomial first integral, quadratic vector fields, phase portraits.

The first and third authors are supported by a MEC grant number MTM 2005-02094. The second author is partially supported by a DGICYT grant number MTM2005-06098-C02-01 and by a CICYT grant number 2005SGR 00550.