# Lines of Principal Curvature on Canal Surfaces in $\mathbb{R}^{3}$ 

RONALDO GARCIA ${ }^{1}$, JAUME LLIBRE ${ }^{2}$ and JORGE SOTOMAYOR ${ }^{3}$<br>${ }^{1}$ Instituto de Matemática e Estatística, Universidade Federal de Goiás Cx. Postal 131, 74001-970 Goiânia, GO, Brasil<br>${ }^{2}$ Departament de Matemàtiques, Universitat Autònoma de Barcelona 08193 Bellaterra, Barcelona, Spain<br>${ }^{3}$ Instituto de Matemática e Estatística, Universidade de São Paulo Rua do Matão, 1010, Cidade Universitária, 05508-090 São Paulo, SP, Brasil

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#### Abstract

In this paper are determined the principal curvatures and principal curvature lines on canal surfaces which are the envelopes of families of spheres with variable radius and centers moving along a closed regular curve in $\mathbb{R}^{3}$. By means of a connection of the differential equations for these curvature lines and real Riccati equations, it is established that canal surfaces have at most two isolated periodic principal lines. Examples of canal surfaces with two simple and one double periodic principal lines are given.


Key words: Riccati equation, principal curvature lines, canal surfaces.

## 1. INTRODUCTION

The study of principal curvature lines, along which a surface in $\mathbb{R}^{3}$ bends extremely and their umbilic singularities was founded by Monge, Dupin and Darboux. See (Gray 1998) and (Sotomayor 2003) for references. For the basic facts about principal curvature lines on surfaces the reader is addressed to (do Carmo 1976), (Spivak 1979) and (Struik 1988).

As a consequence of the work of Monge and Dupin the lines of curvature on quadrics and toroidal - Dupin Cyclides - surfaces were determined. See (Fischer 1986, Chap. 3), for an outline of the theory and for a collection of remarkable illustrations.

