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A Route to Chaos in the Boros–Moll Map

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The Boros–Moll map appears as a subsystem of a Landen transformation associated to certain rational integrals and its dynamics is related to their convergence. In the paper, we study the dynamics of a one-parameter family of maps which unfold the Boros–Moll one, showing that the existence of an unbounded invariant chaotic region in the Boros–Moll map is a peculiar feature within the family. We relate this singularity with a specific property of the critical lines that occurs only for this special case. In particular, we explain how the unbounded chaotic region in the Boros–Moll map appears. We especially explain the main contact/homoclinic bifurcations that occur in the family. We also report some other bifurcation phenomena that appear in the considered unfolding.

Keywords: Boros–Moll map; chaotic set; critical line; homoclinic bifurcation; noninvertible planar map; snapback repellor.

1. Introduction

In this paper, we study some dynamical properties of the one-parameter family of planar maps $G_h : \mathbb{R}^2 \to \mathbb{R}^2$ given by

$$G_h(x, y) := \left(\frac{h(x + y) + xy + 9}{(x + y + 2)^{4/3}}, \frac{x + y + 6}{(x + y + 2)^{2/3}} \right). \quad (1)$$

For $h = 5$, the map G_5 appears as a subsystem on an uncoupled Landen transformation defined in $\mathbb{R}^5$ introduced by Boros and Moll [2000], see also [Boros & Moll, 2001]. Roughly speaking, given a definite integral depending on several parameters, a Landen transformation is a map on these parameters that leaves invariant the integral, see [Moll, 2012, p. 412] for a more precise definition, and [Almkvist & Gert, 1988] for a historical account of