Journal of Difference Equations and Applications
Vol. 16, Nos. 5–6, May–June 2010, 573–595

Singular perturbations of z^n with a pole on the unit circle

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(*Received 16 July 2009; final version received 23 July 2009*)

Dedicated to Robert Devaney on the occasion of his 60th birthday

We consider the family of complex maps given by $f_{\lambda,a}(z) = z^n + \lambda/(z-a)^d$ where n, $d \geq 1$ are integers, and a and λ are complex parameters such that $|a| = 1$ and $|\lambda|$ is sufficiently small. We focus on the topological characteristics of the Julia and Fatou sets of $f_{\lambda,a}$.

Keywords: rational map; iteration; Julia set; Fatou set

AMS Subject Classification: 37F10; 37F40

1. Introduction

In the last few years, a number of papers have appeared that deal with the dynamics of functions obtained by a perturbation of the complex function $z \mapsto z^n$ by adding a pole at the origin [3, 5–7]. These rational functions are of the form $f_\lambda(z) = z^n + \lambda/z^d$. When $|\lambda| \ll 1$, we consider this function as a *singular perturbation* of z^n. The reason for this terminology is that when $\lambda = 0$, the map is z^n and the dynamical behaviour is well understood. When $\lambda \neq 0$, however the degree jumps to $n + d$ and the dynamical behaviour changes significantly. The interest in this type of perturbation arises from the application of Newton's method to find the roots of a family of polynomials that, at one particular parameter value, has a multiple root. At this parameter value, the Newton iteration function undergoes a similar type of singular perturbation.

In [8], we investigated a more general class of functions for which the pole is not located at the origin but rather is located at some other point in the complex plane that does not lie on the unit circle. In particular, we considered the family of functions given by

$$f_{\lambda,a}(z) = z^n + \frac{\lambda}{(z-a)^d}, \tag{1}$$

where $n \geq 2$ and $d \geq 1$ are integers, and a and λ are complex parameters where $|a| \neq 0, 1$ and $|\lambda|$ is sufficiently small.

In this paper, we continue the study of the family $f_{\lambda,a}$. In the first part, we study the dynamics of equation (1) when the pole a is on the unit circle and $|\lambda|$ is sufficiently small.

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ISSN 1023-6198 print/ISSN 1563-5120 online

DOI: 10.1080/10236190903257842
http://www.informaworld.com