DISAPPEARANCE OF LIMIT CYCLES FOR SOME FAMILIES OF PLANAR VECTOR FIELDS

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Abstract. We are interested to study the disappearance of limit cycles for one parameter families of planar vector fields. We consider several examples of these families for which this disappearance can be easily explained by taking a suitable complexification of the real vector field

In order to know the exact number of limit cycles of the differential equations given by one parameter family of planar vector fields

$$\dot{x} = X(x, \lambda) \qquad x \in \mathbf{R}^2, \lambda \in \mathbf{R},$$

we must take into account that the limit cycles can appear or disappear of the real plane when the parameter λ varies. This phenomenon can be studied by considering the bifurcations near critical points, non hyperbolic limit cycles and separatrix cycles.

The aim of this paper is to give several examples for which we can introduce a suitable complexification of the real vector field such that the number of "complex limit cycles" does not depend on λ.

The main idea is that perhaps there is a similar result to the fundamental theorem of algebra on the number of zeros of a polynomial for the number of complex limit cycles of a planar polynomial differential system. This idea has been already stated in many papers (see for instance [**Y1**]).

The complex coordinates that we will usually choose, will not be the usual ones, i.e. $x = (x_1, x_2)$ with $x_1, x_2 \in \mathbf{C}$. They will be the polar coordinates (r, θ) but with r as a complex number. We will say that a trajectory is a *complex limit cycle* if it is a solution of the differential equation in polar coordinates satisfying $r(0) = r(2\pi) \in \mathbf{C}$, and this trajectory is isolated into the set of trajectories with this property.

We begin with two examples that have been already considered in [**Y2**].

Example 1. Consider the system of differential equations

$$(1) \qquad \begin{cases} \dot{x} = -y + x(x^2 + y^2 - \lambda) \\ \dot{y} = x + y(x^2 + y^2 - \lambda). \end{cases}$$