ON POLYNOMIAL SYSTEMS WITH INVARIANT ALGEBRAIC CURVES

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Abstract. For planar polynomial differential equations with some invariant algebraic curves $F_1, \ldots F_k$ and some algebraic curves without contact $F_{k+1}, \ldots, F_m$ we give sufficient conditions to ensure that the differential equation has a Dulac function of type $\prod_{i=1}^{m} F_i^{\alpha_i}$. Using these Dulac functions we obtain results on the maximum number of limit cycles for the differential equation. We apply the results obtained to quadratic differential equations with second order invariant curves.

1. INTRODUCTION.

We start by fixing the notation. In the sequel X will always denote the vector field associated to the planar differential equation under consideration.

We will say that the differential equation

$$\dot{x} = P(x,y),\ \dot{y} = Q(x,y), \tag{1}$$

is *polynomial*, if P and Q are polynomials with real variables (x,y) (that is, $P, Q \in \mathbb{R}[x,y]$). In this case the degree of X is $\max(degP, \deg Q)$.

We recall that for a vector field $X(x,y)$, div$X$ is $\frac{\partial X_1(x,y)}{\partial x} + \frac{\partial X_2(x,y)}{\partial y}$, where X_1, X_2 are the components of X.

If a regular function F is such that divFX does not change sign on a subset $U \subset \mathbb{R}^2$ we will say that F is a *Dulac function* for X on U. It is well known the Bendixson-Dulac Criterion to ensure non existence of limit cycles for planar vector fields X having a Dulac function on a simply connected open subset U.

The polynomial systems of differential equations of degree 2 are usually called quadratic systems. In the sequel, these systems will be called simply QS. The classical proof of the fact that a QS with 2 invariant straight lines has no limit cycles is due to Bautin [2] and uses the general expression $\dot{x} = x(ax + by + c)$, $\dot{y} = y(dx + ey + f)$, for the QS and a Dulac function $F(x,y) = x^{\alpha}y^{\beta}$. As far as we know, in that paper, the existence of invariant curves ($x = 0$ and $y = 0$) was used for the first time to prove non existence of limit cycles. Recently, in [9], the existence of invariant algebraic curves is related with the study of centers for QS. These are the main inspirations of the present paper. Our main goal will be to extend Bautin's techniques to the situation in which the existence of invariant algebraic curves and algebraic solutions without contact is known.

The usual inner product will be represented by $\langle\ ,\ \rangle$. We need the following definitions

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